Name_____

School_____

University of Houston
High School Mathematics Contest
Algebra II Exam – Spring 2012

1. State the number of integer factors of the number 2012.
   a. 3
   b. 4
   c. 6
   d. 12
   e. 24

2. Find the equation of the parabola that passes through the points $(-2,0)$, $(-4,-9)$ and $(0,5)$.
   a. $f(x)=-x^2+3x+5$
   b. $f(x)=x^2-3x-10$
   c. $f(x)=\frac{1}{2}x^2+\frac{3}{2}x+5$
   d. $f(x)=-\frac{1}{2}x^2+\frac{3}{2}x+5$
   e. $f(x)=2x^2-6x-10$

3. If $A+B=2$ and $A^2+B^2=5$, find A^3+B^3.
   a. 11
   b. 8
   c. 10
   d. 5
   e. 14

4. Monica and Lee spent the weekend at the amusement park, and each of them spent too much money in the candy store. The candy store sold large gumballs for 1 quarter, assorted lollipops for 2 quarters and chocolate bars for 4 quarters. While in the park, each child bought a total of 10 pieces of candy. Monica bought twice as many gumballs as Lee, and one-third as many lollipops as Lee. Lee spent a total of $6.25 and Monica spent $5.00 at the candy store. How many total chocolate bars were purchased?
   a. 4
   b. 5
   c. 6
   d. 7
   e. 8

5. Let $f(x) = x^3 - 3x^2 + 3x - 9$. Find the value of x for which $f^{-1}(x) = 0$.
   a. 3
   b. –9
   c. $-3, 3$
   d. $\sqrt{3}$
   e. 0

6. Find the equation of the line that passes through the centers of the following circles:

$$x^2 + y^2 - 8x + 2y = -1$$
$$2x^2 + 2y^2 + 8x - 4y = 62$$

   a. $y = 1$
   b. $x + 3y = -1$
   c. $x + 3y = 1$
   d. $x = 4$
   e. $3x + 8y = 4$

7. Let $f(x)$ be a function on the real numbers with the property that $f(a+b)=f(a)f(b)$ for all real numbers a and b. If $f(5)=8$, find $f(-10)$.
   a. -16
   b. -64
   c. $\frac{1}{8}$
   d. $-\frac{1}{16}$
   e. $\frac{1}{64}$

8. Which of the following numbers is not a perfect square? (Hint: Any positive integer may be written in the form $100a+b$, where a and b are nonnegative integers and $b<100$)
   a. 32913169
   b. 61168041
   c. 92294449
   d. 66797945
   e. 7107556

9. Given the number $N=1001$ written in base b for an arbitrary integer $b>1$. Which of the following statements must be true about N ?
   a. N is divisible by b.
   b. N is divisible by $b-1$.
   c. N is divisible by $b+1$
   d. N is divisible by b^2+1
   e. N is divisible by 2.

10. Two real numbers *x* and *y* have the property that their difference is 6 and the difference of their square roots is 1. Find the sum of *x* and *y*.
    a. 18
    b. $\frac{37}{4}$
    c. $\frac{49}{4}$
    d. $\frac{37}{2}$
    e. $3+\sqrt{85}$

11. The vertices of a parallelogram are the intersection points of pairs of the following lines. Find the perimeter of the parallelogram.

$$x+2y=10 \qquad 3y=4x-7$$
$$x+2y=-1 \qquad -4x+3y=15$$

a. $4\sqrt{5}+10$
b. $10\sqrt{2}+10$
c. $8\sqrt{5}$
d. $4\sqrt{2}+10$
e. $2\sqrt{53}+2\sqrt{37}$

12. Find the sum of the absolute values of all solutions to the following equation:
$\left|\,|x+1|-2\,\right|=4$.

a. 16
b. 12
c. 4
d. 2
e. 0

13. Let $a=\sqrt[3]{9+4\sqrt{5}}$ and $b=\sqrt[3]{9-4\sqrt{5}}$. Which of the following rational numbers is equal to $a+b$?

a. 3
b. 6
c. $\frac{9}{2}$
d. $\frac{27}{10}$
e. $\frac{18}{5}$

14. If $m = 1.46666\ldots$ and $n = 0.407407\overline{407}\ldots$, find $\frac{m}{n}$.

    a. $\frac{66}{185}$

    b. $\frac{18}{5}$

    c. $\frac{36}{11}$

    d. 3

    e. $\frac{120}{37}$

15. Jon and Quinn are learning about gravity. Jon drops a ball from the top of a 225 ft tall building and observes when it hits the ground. Quinn is across the street at the top of an 180 ft tall building, throws a ball up in the air and watches it fall to the ground. The balls left the boys hands at the same time. The following equations describe the height of each ball, H (in feet), at time *t* seconds after each ball was released.

$$\text{Jon: } H(t) = -16t^2 + 225$$

$$\text{Quinn: } H(t) = -16t^2 + 12t + 180$$

    Which of the following describe the observed impact of the balls?

    a. Jon's ball hits the ground 1 second before Quinn's ball hits the ground.
    b. Quinn's ball hits the ground 45 seconds before Jon's ball hits the ground.
    c. Both balls hit the ground at the same time.
    d. Quinn's ball hits the ground $3/4$ second before Jon's ball hits the ground.
    e. Jon's ball hits the ground $3/4$ second before Quinn's ball hits the ground.

16. Betty and Sam are stuffing envelopes for their favorite political candidate. Together, Betty and Sam can complete the task in 6 hours. If Sam stuffed all the envelopes by himself, it would take him 10 hours to finish the task. How many hours would it take Betty to do the task by herself?

    a. 12
    b. 13
    c. 14
    d. 15
    e. 16

17. Find the area of the region of the coordinate plane that is the solution to the following system of inequalities:

$$-2 \le x \le 2$$
$$-3 \le y \le 3$$
$$6 \ge x - 4y$$
$$x + y \le 1$$

a. 6
b. 8
c. 10
d. 12
e. 14

18. Find the intersection point of the graphs of the functions $f(x) = 2^{x+7}$ and $g(x) = 8^{2x-1}$.

a. $(0, 64)$
b. $(8, 32768)$
c. $(2, 512)$
d. $(0, 2)$
e. $(0, 8)$

19. At 8:15 am, the line at the passport office was already long when the first clerk helped the first customer. By 9:00 am, 30% of the people who had been in line since 8:15 had been helped. Unfortunately, more people had come in to the office and the line was now 20% longer than it had been at 8:15. By 11:00 am, all of the people in the original line had been helped, but the line was now 50% longer than it had been at 9:00 am. The passport office closed the line at 11:00 am and served customers at the rate of 9 per hour from 11:00 am until the last customer was finished at 5:00 pm. How many customers got in line between 8:15 am and 9:00 am?

a. 6
b. 10
c. 15
d. 20
e. 36

20. Solve for x: $36^{x} - 6^{x+1} = 16$

    a. $\log_6 2$

    b. $3\log_6 2$

    c. $2\log_6 3$

    d. $\log_2 6$

    e. $\log_6 16$

21. Which of the following is a quadratic function with a root of $2+i$ and a *y*-intercept of 10?

    a. $f(x) = x^2 - 4x + 10$

    b. $f(x) = -2x^2 + 8x + 10$

    c. $f(x) = 2x^2 + 8x + 10$

    d. $f(x) = 2x^2 - 4x + 10$

    e. $f(x) = 2x^2 - 8x + 10$

22. Assume *b*, *x* and *y* are positive integers and *b* > *1*. If $\log_b\left(x^2\right) = 16$ and $\log_b\left(y^3\right) = 18$, solve for y in terms of x.

    a. $y = x^{3/4}$

    b. $y = x^{2/3}$

    c. $y = \sqrt[3]{\frac{8}{9}}\, x^{2/3}$

    d. $y = x^{3/8}$

    e. $y = x^{1/8}$

23. Given $f\left(\sqrt{\frac{x-2}{x+1}}\right)=\frac{3x+4}{5x+6}$, find $f(2)$.

a. $\frac{5}{8}$

b. $\frac{4}{7}$

c. 0

d. $\frac{2}{3}$

e. $\frac{1}{2}$

24. Which of the following is the solution to the inequality $x^3-1<13x-13$?

a. $-4<x<3$

b. $x<-4$ or $1<x<3$

c. $x<-3$

d. $-4<x<1$ or $x>3$

e. $x<3$

25. The expression below is equal to a rational number $\frac{a}{b}$, where a and b have no common factors. Find $a+b$.

$$\frac{\frac{2}{3}-\frac{5}{18}}{\frac{10}{24}+\frac{20}{36}}$$

a. 7

b. 37

c. 72

d. 84

e. 98

26. The force of gravity between two objects is inversely proportional to the square of the distance between them. Two objects, *A* and *B*, are 10,000 km apart and the force of gravity between them is F newtons. Suppose the objects move and the force between them is now 16F newtons. How far apart are the objects after they moved?
    a. 40,000 km
    b. 160,000 km
    c. 5000 km
    d. 2500 km
    e. 625 km

27. Li bought a bag of jelly beans. If he eats 6 red jelly beans and then randomly selects a seventh jelly bean, the probability that the randomly selected bean is green is 0.15. If he were to add nine green jelly beans to the original bag and then select a bean at random, the probability the bean is not green would be 0.8. If one jelly bean is randomly drawn from the original bag, what is the probability that it is not green?
    a. $\frac{7}{8}$
    b. $\frac{6}{7}$
    c. $\frac{5}{6}$
    d. $\frac{17}{20}$
    e. $\frac{3}{4}$

28. A right triangle has sides of lengths c, c^2, c^3 for a real number $0 < c < 1$. Find the value of c.
    a. $\frac{1+\sqrt{5}}{2}$
    b. $\frac{\sqrt{2\sqrt{5}-2}}{2}$
    c. $\sqrt{\sqrt{5}-1}$
    d. $\frac{\sqrt{2\sqrt{5}+2}}{2}$
    e. $\frac{\sqrt{2-2\sqrt{5}}}{2}$