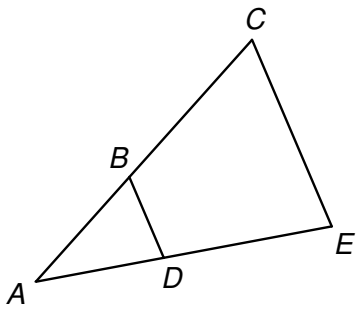**Name:** _____   **School:** _____

University of Houston High School Mathematics Contest
Geometry Exam – Spring 2016

Answer the following. Note that diagrams may not be drawn to scale.

1. In the figure below, $\overline{BD} \parallel \overline{CE}$, $AB = 3$, $BD = 4$ and $CE = 20$. Find the length of $\overline{BC}$.


(A) 16

(B) 15

(C) 12

(D) 48

(E) 19

2. If the length of a rectangle is increased by 40%, and its width is decreased by 30%, how is the area of the rectangle affected?

(A)   The area decreases by 2%.
(B)   The area increases by 10%.
(C)   The area increases by 12%.
(D)   The area increases by 0.88%.
(E)   The area decreases by 7%.

3. The figure below consists of eight nonoverlapping polygons numbered 1 through 8, where Polygons 1-7 are squares. If the area of Polygon 2 is 16 and the area of Polygon 6 is 9, find the area of Polygon 8.
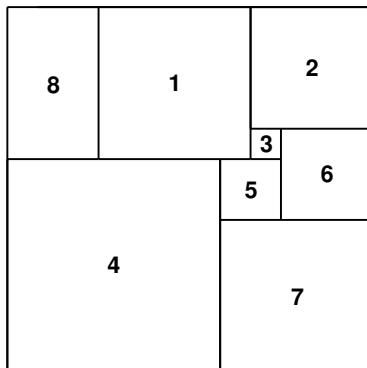
(A) 12

(B) 15

(C) 20

(D) 8

(E) 10

4. Which of the following figures are always rotationally and reflectionally symmetric?

      I. Kite   II. Parallelogram   III. Regular Pentagon

      IV. Rhombus   V. Isosceles Trapezoid

    A. II, III and IV only
    B. II and IV only
   C. I, II, III, IV and V
    D. III and IV only
    E. IV only

5. Given any conditional statement, the inverse of the contrapositive of the conditional statement is also known as the _____.

    A. conditional   B. biconditional   C. contrapositive   D. inverse   E. converse
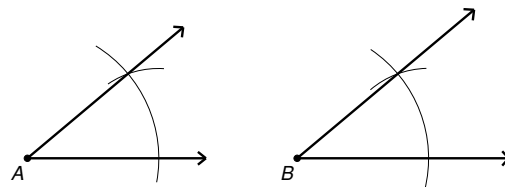
6. Mrs. Rhoades is teaching her class how to construct an angle congruent to a given angle.

   She begins by drawing angle A:
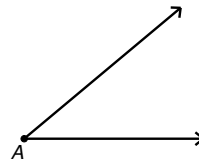
    She draws a ray originating at point B, and uses her compass to make the markings shown at the right.
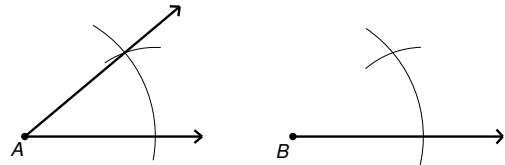
    She completes the construction by drawing a ray from point B through the intersection of the two compass markings at the right.

    Mrs. Rhoades wants the students to understand why $\angle A \cong \angle B$, and she explains that they are corresponding parts of congruent triangles. She adds labels and segments to the diagram as shown at the right.
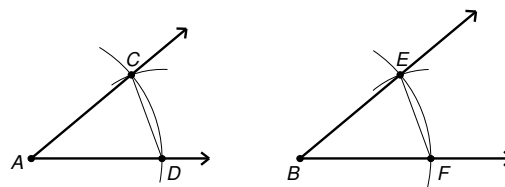
    Based strictly on Mrs. Rhoades' construction, by what theorem or postulate is $\triangle ACD$ congruent to $\triangle BEF$?

    A. Angle-Side-Angle   B. Side-Angle-Side   C. Angle-Side-Side
    D. Side-Side-Side   E. Angle-Angle-Side

7.   In the figure below, $KM = 40$ and $JL = 20$. Find the length of $\overline{AC}$.
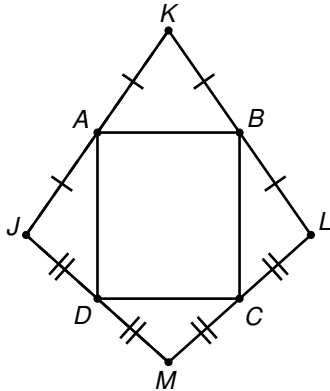
(A) $10\sqrt{5}$

(B) $20\sqrt{2}$

(C) $20\sqrt{3}$

(D) $10 + 10\sqrt{3}$

(E) $10 + 5\sqrt{2}$

8.   Kathy lives 120 miles away from Erin and 40 miles away from Mark. Let x represent the distance between Erin's house and Mark's house. If the houses belonging to Kathy, Erin and Mark do not all lie on the same line on a map, which of the following inequalities shows the possible values of x?

(A) $40 < x \le 40\sqrt{10}$ (B) $0 < x < 160$ (C) $80 < x < \infty$

(D) $80 < x < 160$ (E) $0 < x < 40\sqrt{10}$

9.   Suppose that C is the midpoint of $\overline{AB}$. Given coordinates $A\left(-\dfrac{2}{5}, \dfrac{1}{6}\right)$ and

$C\left(-\dfrac{2}{35}, \dfrac{13}{48}\right)$, find the coordinates of B.

(A) $\left(-\dfrac{8}{35}, \dfrac{7}{32}\right)$

(B) $\left(\dfrac{11}{15}, \dfrac{407}{840}\right)$

(C) $\left(-\dfrac{6}{35}, -\dfrac{5}{96}\right)$

(D) $\left(\dfrac{2}{7}, \dfrac{3}{8}\right)$

(E) $\left(\dfrac{12}{35}, \dfrac{5}{48}\right)$

10. Consider a unit cube which measures one centimeter on each side. Let V_1 represent the volume of the largest cone which fits inside a unit cube, and let V_2 represent the volume of the largest sphere which fits inside a unit cube. Find the ratio $V_1 : V_2$.

(A) $3:2$ (B) $1:12$ (C) $2:3$ (D) $1:4$ (E) $1:2$

11. A construction is performed on triangle ABC, resulting in the markings shown below. Based on the construction markings, we can conclude that $\overline{BD}$ is a(n):
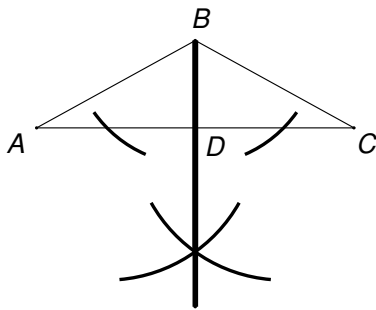
I. perpendicular bisector of $\overline{AC}$

II. median of $\triangle ABC$

III. altitude of $\triangle ABC$

IV. bisector of $\angle B$

(A) II only   (B) I, II and III only   (C) III only   (D) IV only   (E) I, II, III and IV

12. Aidan owns a set of rectangular tiles that can magnetically click together at their sides. Aidan has 1 blue tile, 3 green tiles, and 4 yellow tiles. The blue tile measures $6'' \times 3''$, the green tiles each measure $4'' \times 3''$, and the yellow tiles each measure $2'' \times 3''$. He arranges the tiles into a rectangular "train" of tiles; one such arrangement is shown below.
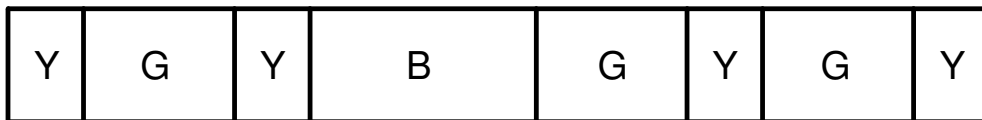
Aidan then plays a game as follows: His train becomes a rectangular dartboard on the floor. He steps back and throws a dart at his rectangular train, removes the tile that is hit by the dart, and clicks the remaining pieces together to form a new rectangular train of tiles. He continues this process of throwing the dart, removing the tile that is hit and clicking the remaining pieces together. If Aidan begins a game with all eight tiles, what is the probability that he removes a yellow tile and then a green tile and then a yellow tile in his first three throws of the dart? Assume that the dart is equally likely to land at any point on the arranged magnetic tiles, and that all removed tiles are set aside for the entirety of the game.
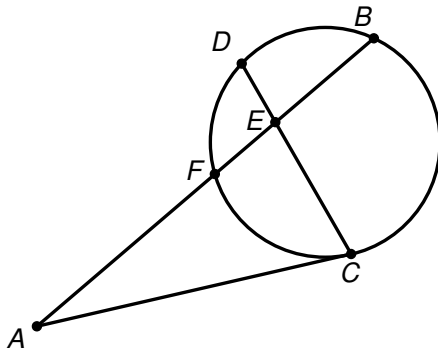
(A) $\dfrac{4}{65}$ (B) $\dfrac{96}{2197}$ (C) $\dfrac{3}{28}$ (D) $\dfrac{1}{7}$ (E) $\dfrac{3}{65}$

13. A regular decagon is shown below with ten parallelograms drawn inside. Find the indicated angle measure, x.
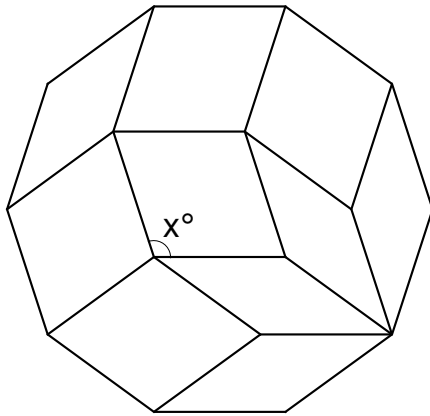
(A) 108

(B) 120

(C) 144

(D) 150

(E) 112

14. In the figure below, $\overline{AB}$ is a secant of the circle, $\overline{AC}$ is a tangent segment, and $\overline{FB}$ and $\overline{DC}$ are chords. Find the measure of $\angle A$ using the given information below.

Given:

$\overparen{DF}$ measures $30°$

$m\angle BEC = 100°$

$\overparen{DB}$ measures $x°$

$\overparen{FC}$ measures $(x+40)°$

(A) $10°$ (B) $5°$ (C) $35°$ (D) $85°$ (E) $40°$

15. A craftsman creates the lateral surface of a cone from a thin piece of metal so that the cone is hollow (like an ice cream cone). A square pyramid of the same height is placed inside the cone, and has been crafted so that the square base of the pyramid is inscribed in the circular base of the cone. The slant height of the pyramid is 9 inches and the height of the pyramid is 3 inches. Find the volume of the cone in cubic inches.

(A) 144π

(B) 72π

(C) 288π

(D) 576π

(E) 432π

16.

Jill and Kayla's houses are shown at the right, along with a diagram of the roads near their houses. A dotted line is shown in the middle of each road, and A, B, C, D and E are the points where the dotted lines intersect.
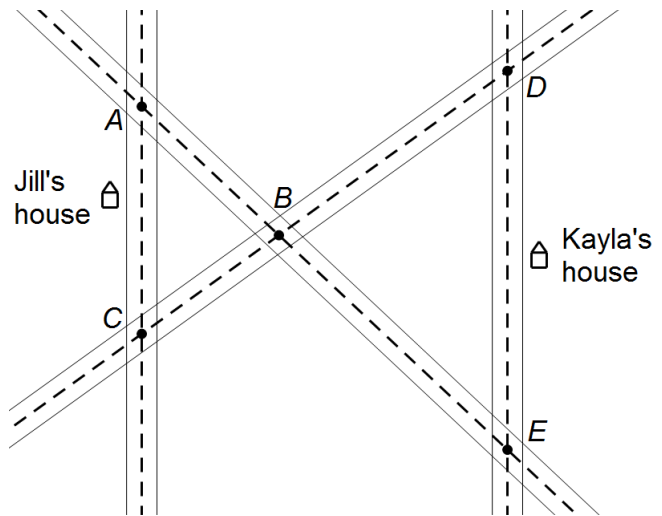
The diagram at the right is a simplified version of the sketch above, where the segments represent the center lines of the roads. Points J and K represent points directly in front of Jill's and Kayla's houses, respectively.
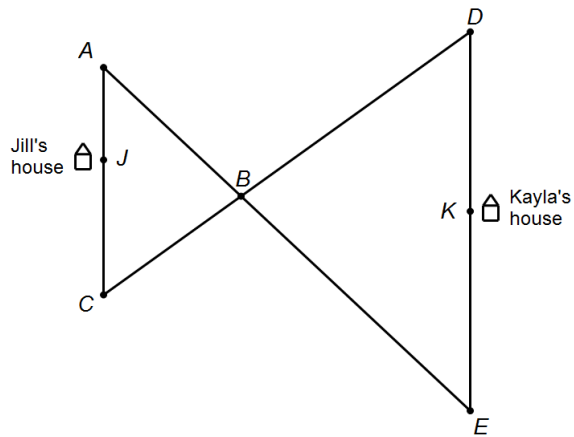
Jill and Kayla live on parallel streets. Several important distances and relationships are given below the diagram.

What is the shortest distance in miles from Jill's house to Kayla's house? (Compute distances based on the center lines of each road.)

Given Information: (All units are in miles.)
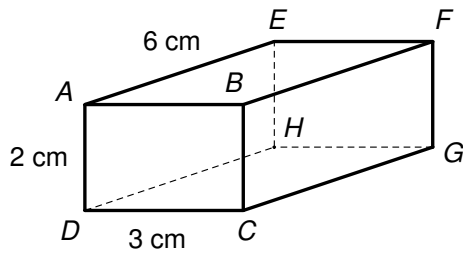
$$AB = 9 \qquad BD = 8 \qquad BE = 12$$

$$AJ = \frac{2}{3}DK \qquad JC = AJ + 2 \qquad KE = DK + 1$$

(A) 29.5   (B) $17\frac{13}{22}$ (C) 28.5   (D) $18\frac{13}{22}$ (E) 26

17.   A triangle has vertices $A(-5, 6)$, $B(-5, -4)$, and $C(3, 4)$. Find the radius of the circle which is circumscribed about $\triangle ABC$.

(A) 5   (B) $4\sqrt{2}$ (C) $\sqrt{34}$ (D) $\dfrac{4\sqrt{13}}{3}$ (E) $3\sqrt{5}$

18. A rectangular prism is shown below with measurements as shown. Find the measure of $\angle DFG$.

(A) $\tan^{-1}\left(\dfrac{2\sqrt{5}}{15}\right)$ (B) $\sin^{-1}\left(\dfrac{2}{7}\right)$ (C) $\tan^{-1}\left(\dfrac{1}{3}\right)$ (D) $\tan^{-1}(3)$ (E) $\cos^{-1}\left(\dfrac{2}{7}\right)$

19. A unique dartboard is shown below. ABCDEF is a regular hexagon with side length 12 cm, and six of its diagonals are drawn to form a star-shaped pattern. If a dart is thrown and is equally likely to land anywhere within JKLM, find the probability that the dart lands within the shaded inner hexagon.
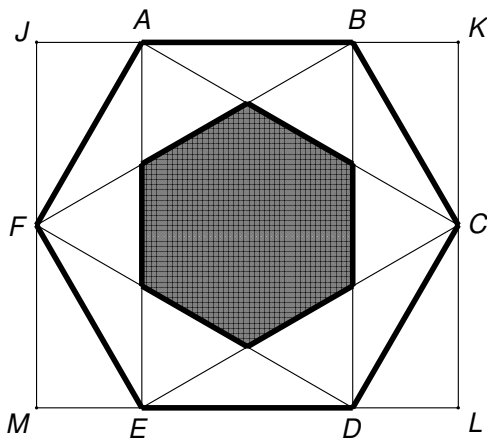

(A) $\dfrac{1}{4}$ (B) $\dfrac{\sqrt{3}}{8}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{\sqrt{3}}{6}$ (E) $\dfrac{3}{16}$

20. A right triangle has legs measuring 15 cm and 20 cm. The triangle is rotated about its hypotenuse, tracing out a three-dimensional solid. Find the total surface area of the rotational solid which is formed.

(A) 150 cm^2

(B) 1200π cm^2

(C) 600π cm^2

(D) 420π cm^2

(E) 900π cm^2

21. A cylindrical spice jar holds 0.75 ounces of salt when filled. A larger cylindrical container has a base area which is eight times as large as the base area of the spice jar, and a lateral area which is six times as large as the lateral area of the spice jar. How many ounces of salt would it take to fill the larger container?

(A) 36 (B) 288 (C) $3\sqrt{3}$ (D) $9\sqrt{2}$ (E) $6\sqrt{6}$

22. A windmill-shaped design is shown below, where $AB = 6 \text{ cm}$, $\overset{\frown}{AC}$ has degree measure $16°$, and the arclength of $\overset{\frown}{AC}$ is $\dfrac{4\pi}{5}$ cm. Find the area of the shaded region in square centimeters.

(A) $\dfrac{108\pi}{5}$ (B) 36π (C) 45π (D) 32π (E) 40π
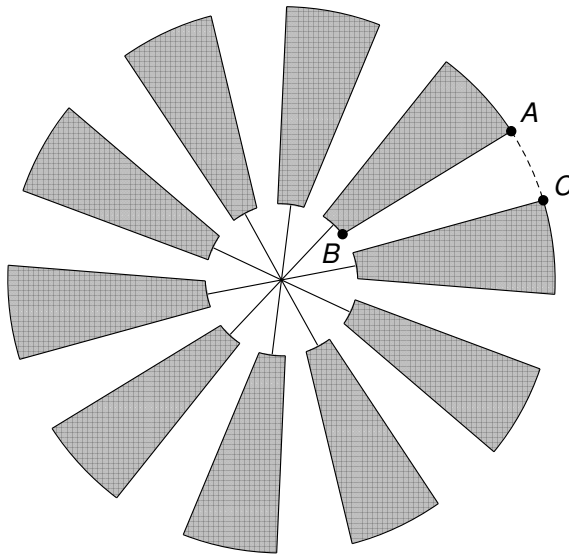
23. The slope of line m is $\dfrac{3x-3}{x+5}$ and the slope of line n is $\dfrac{2x-14}{x+3}$.

If $m \perp n$, which of the following could represent the slope of line n?

(A) $-\dfrac{7}{19}$ (B) $-\dfrac{1}{3}$ (C) $\dfrac{3}{4}$ (D) $-\dfrac{3}{2}$ (E) None of these

24. Consider a triangle in spherical geometry (on the surface of a sphere) which is formed by the minor arcs of three great circles. The sum of the measures of the angles of such a triangle is _____.

(A) equal to 180°

(B) greater than 180° and less than 360°

(C) greater than 180° and less than 540°

(D) greater than 0° and less than 180°

(E) greater than 360° and less than 1080°

25. If the letters A, B, C, and D are used the label the four distinct points on the segment below (not necessarily in that order), what is the probability that B is between A and C? Assume that each arrangement of A, B, C, and D is equally likely.

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{12}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{1}{6}$

26. In the figure below, $\triangle ABC$ is inscribed in circle P, $\overline{BD}$ is an altitude of $\triangle ABC$, and $\overline{PL} \perp \overline{BC}$. If $AB = 12$ and the ratio of $DC : AC$ is 5 to 9, find the area of quadrilateral $BDPL$.
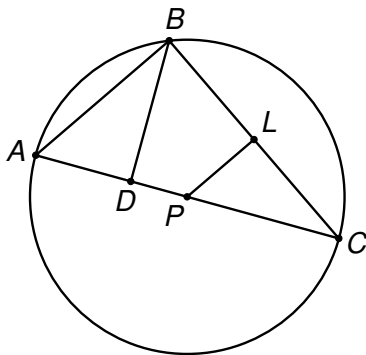
(A) $20\sqrt{2} - \dfrac{45}{2}$ (B) $11\sqrt{5}$ (C) $9\sqrt{5} + 3$ (D) $47\sqrt{5}$ (E) $\dfrac{135}{2} - 16\sqrt{2}$

27. Suppose that ΔABC is dilated by a factor of 250% with center of dilation $(-9, 4)$. If the coordinates of the image are $A'(-4, -6)$, $B'(6, -1)$, and $C'(11, -16)$, find the coordinates of the vertices of ΔABC.

(A) $A\left(-5\frac{2}{3}, -2\frac{2}{3}\right)$, $B\left(1, \frac{2}{3}\right)$, $C\left(4\frac{1}{3}, -9\frac{1}{3}\right)$

(B) $A(-1.5, 19)$, $B(13.5, 11.5)$, $C(21, 34)$

(C) $A(3.5, 29)$, $B(28.5, 16.5)$, $C(41, 54)$

(D) $A(-7, 0)$, $B(-3, 2)$, $C(-1, -4)$

(E) $A(-5, -1)$, $B(-1, 1)$, $C(1, -5)$

Questions 28-30 are write-in answers rather than multiple choice. Write the answer to each question on the answer sheet in the space provided. All irrational answers (containing radicals or π) should be left as exact answers rather than decimal approximations. All fractions should be written in simplest form, with no radicals in the denominator. Do not write any units on the answer sheet.

28. The equation of line ℓ is $10x - 3y = -300$. How many lines are perpendicular to line ℓ, have a y-intercept that is an integer, and intersect line ℓ in the 2nd quadrant?

29. In the figure below, circles A, B, C and D are tangent at point P; circles D, E, F, and G are tangent at point Q; $AB = 2$ and $\overline{JK} \perp \overline{PQ}$. Find the area of the shaded region, in square units.
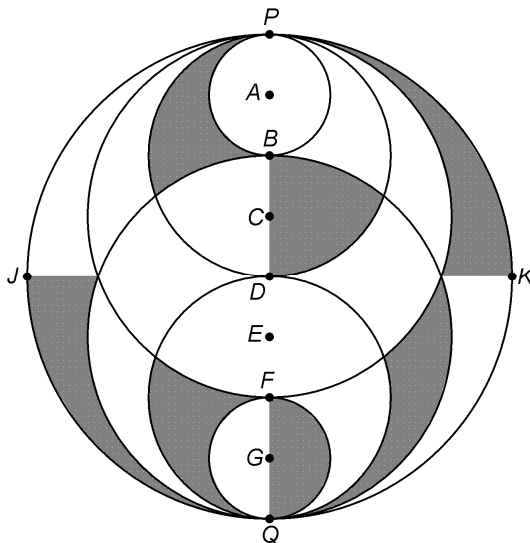
30. Circle Q has equation $x^2 + y^2 - 2x - 6y - 10 = 0$. Find the equation of the line which is tangent to the circle, has negative slope, and passes through the point $(1, -7)$. Write your answer in slope-intercept form.

END OF EXAM ☺