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University of Houston Mathematics Contest: Pre-Calculus Exam 2017

1. The graph of the function $f(x) = 2(x-1)^2 - 8$
   A. has axis of symmetry $x = -1$.
   B. intersects the $x-$ axis at $x = -1$ and $x = 3$.
   C. does not intersect the $x-$ axis.
   D. has vertex $(1,8)$.
   E. does not cross the $y-$ axis.
   F. None of these

2. Find all roots of $g(x) = 2x^3 - x^2 + 8x - 4$.
   A. $x = \frac{1}{2},\ \pm 2i$
   B. $x = \frac{1}{2},\ \pm 2$
   C. $x = \frac{1}{2}i,\ \pm 2$
   D. $x = 2,\ \pm 2i$
   E. $x = 2,\ \pm i$
   F. None of these

3. Given $f(x) = \frac{3x-5}{x-2}$, find $f^{-1}(2)$.
   A. Not possible.
   B. 1
   C. -2
   D. 0
   E. $\frac{1}{2}$
   F. None of these

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4. Solve $\log_4(x^2-2)-\log_4(x+1)=1$.

   A. $x=2\pm\sqrt{10}$
   B. $x=1,6$
   C. $x=6$
   D. $x=2+\sqrt{10}$
   E. $x=\sqrt{2}$
   F. None of these

5. Suppose that $\sin\theta=\frac{3}{7}$ and $\tan\theta<0,$ find $\sec\theta$.

   A. $-\frac{2\sqrt{10}}{7}$
   B. $\frac{7}{2\sqrt{10}}$
   C. $-\frac{7}{2\sqrt{10}}$
   D. $-\frac{7}{3}$
   E. $-\frac{3}{2\sqrt{10}}$
   F. None of these

6. Give the exact value of $\sin\left(\frac{5\pi}{12}\right)$.

   A. $\frac{1}{4}$
   B. $\frac{\sqrt{6}+\sqrt{2}}{4}$
   C. $\frac{\sqrt{6}-\sqrt{2}}{4}$
   D. $\frac{\sqrt{3}+\sqrt{2}}{4}$
   E. $\frac{\sqrt{2}-\sqrt{3}}{4}$
   F. None of these

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7. Find $\sec(2\alpha)$ if $\csc(\alpha)=\frac{5}{4}$.

   A. $-\frac{25}{7}$

   B. $\frac{24}{25}$

   C. $-\frac{7}{25}$

   D. $\frac{25}{24}$

   E. $\frac{10}{3}$

   F. None of these

8. If the sum of the first ten terms of an arithmetic sequence is 163 and the sum of terms 11 through 15 is 243, what is the first term of the sequence?

   A. $\frac{412}{25}$

   B. $-\frac{7}{5}$

   C. $-\frac{77}{25}$

   D. $\frac{2}{5}$

   E. $\frac{323}{75}$

   F. None of these

9. $\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-12x+1}\right)=$

   A. 0

   B. $\frac{1}{3}$

   C. 6

   D. 2

   E. Does not exist

   F. None of these

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10. An expression equivalent to $\sqrt{3}\sin x + \cos x = 1$ is

I. $\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

II. $\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$

III. $\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

IV. $\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

A. I only
B. II only
C. III only
D. IV only
E. I and III only
F. II and IV only
G. III and IV only
H. None of these

11. Give the general solution to $\tan^2(2x) - 2\sec^2(2x) + 3 = 0$.

A. $x = \frac{\pi}{8}k;\ k \in \mathbb{Z}$

B. $x = \frac{\pi}{4}k;\ k \in \mathbb{Z}$

C. $x = \frac{\pi}{8}k + 1;\ k \in \mathbb{Z}$

D. $x = \frac{\pi}{4}k + \frac{\pi}{8};\ k \in \mathbb{Z}$

E. $x = \frac{\pi}{4}k + \frac{\pi}{2};\ k \in \mathbb{Z}$

F. None of these

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12. The graph of the function $f(x)=\dfrac{3x^2+12x+12}{2x^2-3x+1}$ has a horizontal asymptote. If the graph crosses this asymptote, give the $x-$coordinate of the intersection. Otherwise, state that the graph does not cross the asymptote.

A. $-\dfrac{6}{11}$

B. $-\dfrac{7}{11}$

C. $-\dfrac{10}{11}$

D. $\dfrac{10}{11}$

E. The graph does not cross the asymptote.

F. None of these

13. An algebraic expression equivalent to $\cos\left(\tan^{-1}\left(\cos\left(2\sin^{-1}(x)\right)\right)\right)$ is

A. $\dfrac{1}{\sqrt{4x^4-4x^2+2}}$

B. $-\dfrac{1}{2x^2-1}$

C. $1-2x^2$

D. x

E. $\dfrac{1-2x^2}{\sqrt{4x^4-4x^2+2}}$

F. None of these

14. If $f(2x+3)=x^2-1$, find $f(x-4)$.

A. $\dfrac{1}{4}x^2-\dfrac{7}{2}x+\dfrac{45}{4}$

B. $\dfrac{1}{4}x^2+\dfrac{1}{2}x-\dfrac{3}{4}$

C. $x^2-8x+15$

D. $4x^2-20x+24$

E. $\dfrac{1}{4}x^2-\dfrac{1}{2}x-\dfrac{5}{4}$

F. None of these

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15. Find the scalars α and β such that $\vec{c} = \alpha\vec{a} + \beta\vec{b}$ where $\vec{a} = -\vec{i} + 2\vec{j}, \vec{b} = 3\vec{i} - 5\vec{j}$, and $\vec{c} = \vec{i} + 3\vec{j}$.
    A. $\alpha = -1, \beta = 4$
    B. $\alpha = 4, \beta = -1$
    C. $\alpha = -14, \beta = 11$
    D. $\alpha = -3, \beta = 4$
    E. $\alpha = 14, \beta = 5$
    F. None of these

16. If each term of a sequence has explicit formula $a_n = 2^n + 3^n$, a recursive formula for the same sequence could be
    A. $a_{n+1} = 5a_n - 6$
    B. $a_{n+2} = a_{n+1} \cdot a_n - 30$
    C. $a_{n+2} = 5a_{n+1} - 6a_n$
    D. $a_{n+2} = 5a_{n+1} + 6a_n$
    E. $a_{n+1} = 2a_n + 3$
    F. None of these

17. In a bacteria-growing experiment, a biologist observes that the number of bacteria in a certain culture triples every 4 hours. After 20 hours, it is estimated that there are 1 million bacteria in the culture. How many bacteria were present initially (round to the nearest whole number)?
    A. 4315
    B. 4115
    C. 3915
    D. 4726
    E. 3370
    F. None of these

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18. Give the polar form for the complex number $z = -1 + i\sqrt{3}$.

A. $z = 2\cos\left(\frac{\pi}{3}\right) + 2i\sin\left(\frac{\pi}{3}\right)$

B. $z = 2\cos\left(\frac{\pi}{3}\right) - 2i\sin\left(\frac{\pi}{3}\right)$

C. $z = 2\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

D. $z = 2\cos\left(\frac{2\pi}{3}\right) + 2i\sin\left(\frac{2\pi}{3}\right)$

E. $z = \cos\left(\frac{\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)$

F. None of these

19. A circular pizza is cut into 8 equal slices. The outer edge of one of the crusts has a length of 11.5 cm. To the nearest centimeter, what is the length of the diameter of this pizza?

A. 29 cm
B. 16 cm
C. 58 cm
D. 21 cm
E. 32 cm
F. None of these

20. A triangle has sides of $5, 7,$ and 11. Find the length of the altitude to the longest side.

A. $\frac{3}{4}\sqrt{199}$

B. 7

C. $\frac{3}{22}\sqrt{299}$

D. $\frac{5}{2}\sqrt{3}$

E. $\frac{2}{11}\sqrt{271}$

F. None of these

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21. Give the rectangular form of the polar equation $r = \dfrac{2}{1+2\sin\theta}$.

A. $x^2 + y^2 + 2y = 2$

B. $x^2 - 3y^2 + 8y = 4$

C. $x^2 + 2y^2 + 5y = 3$

D. $x^2 + 5y^2 = 2$

E. $3x^2 - y^2 - 8x = -4$

F. None of these

22. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{2k^2}{n^3} =$

A. 2

B. $\dfrac{2}{3}$

C. $\dfrac{1}{3}$

D. 0

E. Does not exist

F. None of these

23. Solve for x: $4^{x+1} + 2^x = 6$

A. $x = \log_2(\sqrt{97} - 1) - 3$

B. $x = \log_2(\sqrt{97} + 1) - 2$

C. $x = \log_2(3) - 2$

D. $x = \log_2(5) - 2$

E. $x = \log_2(\sqrt{87} - 1) - 2$

F. None of these

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24. Find the area of triangle SAM if $\measuredangle S = 30^o$, $AS = 4$ and $AM = 6$.

A. $4\sqrt{13-6\sqrt{3}}$

B. $2\sqrt{13-3\sqrt{3}}$

C. $4\sqrt{3}+8\sqrt{2}$

D. $4\sqrt{7}$

E. $2\sqrt{3}+4\sqrt{2}$

F. None of these

25. Solve the system for x and y: $\left\{ \begin{array}{c} 2e^{2x}+3e^{y} = 5 \\ -5e^{2x}+2e^{y} = -1 \end{array} \right\}$

A. $x = \ln\sqrt{\frac{13}{19}},\ y = \ln\left(\frac{23}{19}\right)$

B. $x = \ln\left(\frac{11}{19}\right),\ y = \ln\left(\frac{27}{19}\right)$

C. $x = \ln(3),\ y = \ln\left(\frac{13}{9}\right)$

D. $x = \ln\sqrt{\frac{7}{19}},\ y = \ln\left(\frac{27}{19}\right)$

E. No solution

F. None of these

26. Give the partial fraction decomposition of $\frac{2x^2+1}{x^3-6x^2+11x-6}$

A. $\frac{3}{x-1}+\frac{9}{2x-6}-\frac{7}{x-2}$

B. $\frac{3}{x-1}+\frac{5}{x-3}-\frac{1}{x-2}$

C. $\frac{3}{(x-1)^2}+\frac{5}{x-1}-\frac{9}{x-2}$

D. $\frac{3}{2x-2}+\frac{19}{2x-6}-\frac{9}{x-2}$

E. $\frac{3}{(x-1)^2}+\frac{5}{2x-2}-\frac{9}{x-2}$

F. None of these

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27. Express the curve by an equation in x and y given $x(t)=\sec^2(t)+1,\ y(t)=2-\tan(t)$ for $0\le t<\frac{\pi}{2}$.

A. $x=(2-y)^2$

B. $x=(y-2)^2+2$

C. $(x-1)^2+(2-y)^2=1$

D. $(x-1)^2-(y-2)^2=1$

E. $y^2-2=(x-1)^2$

F. None of these

28. Which of the following is equivalent to $\frac{1-\sqrt[3]{x}}{x-1}$?

A. $\frac{1-x}{\sqrt[3]{x^2}-\sqrt[3]{x}}$

B. $\frac{1+x}{\sqrt[3]{x^2}-\sqrt[3]{x}+1}$

C. $\frac{-1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$

D. $\frac{1-x}{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}$

E. $\frac{1+x}{\sqrt[3]{x^2}+2\sqrt[3]{x}+1}$

F. None of these

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29. Give the domain of $g(x) = \arcsin\left(\ln(1-3x)+1\right)$

A. $\left[\frac{1}{3} - \frac{1}{3}e^{\pi/2-1}, \infty\right)$

B. $\left[\frac{1}{3}, \infty\right)$

C. $\left[0, \frac{1}{3} - \frac{1}{3e^2}\right]$

D. $\left(-\infty, \frac{1}{3} - \frac{1}{3e^2}\right]$

E. $\left(-\infty, \frac{1}{3}\right)$

F. None of these

30. Suppose that p and x are positive and that $q = p^4$. Express $p^{log_q x + log_p x}$ in simplest terms of x.

A. $x\sqrt{x}$

B. $2x$

C. x^2

D. $x^2\sqrt[4]{x}$

E. x

F. None of these