

Precalculus Exam - University of Houston Math Contest 2024

- Given: $P(3x - 2) = 3x^2 - 5x + 7$ find $P(7)$.
 - 21
 - 25
 - 27
 - 23
 - 19
 - none of the other answer choices provided

- If $\cos(40^\circ) = b$, what is $\sin(50^\circ)$ in terms of b ?
 - $1/b$
 - $b\sqrt{2}$
 - b
 - $90 - b$
 - $-b$
 - none of the other answer choices provided

- If $\tan(y) = k$, what is $\sin(y)$ in terms of k ?
 - $1/\sqrt{k^2 + 1}$
 - $k/\sqrt{k^2 - 1}$
 - $1/\sqrt{1 - k^2}$
 - $k/\sqrt{k^2 + 1}$
 - $2k$
 - none of the other answer choices provided

4. Which of the following is equivalent to the expression

$$\cos(x) + \cos(90^\circ - x) + \sin(x) + \sin(90^\circ - x)$$

for any angle x ?

- (a) 0
- (b) $2\sin(x)$
- (c) $2(\sin(x) + \cos(x))$
- (d) $2\cos(x)$
- (e) 1
- (f) none of the other answer choices provided

5. If the graph of $f(x) = (m + 3)x^4 + 2x^{n+6} - 1$ is a line then what is the value of $m + n$?

- (a) -3
- (b) -8
- (c) 0
- (d) 3
- (e) 2

Correction: The word "line" should have said "non-horizontal line."
(The 2024 contest standings were not affected by this oversight.)

- (f) none of the other answer choices provided

6. What is the positive value of

$$x + \frac{1}{x}$$

given that

$$\left(x + \frac{1}{x}\right)^2 - x - \frac{1}{x} = 6?$$

- (a) 4
- (b) 6
- (c) 1
- (d) 8
- (e) none of the other answer choices provided
- (f) 3

7. If $f(x) = \sin(x)$ and $g(x) = \cos(x)$ what is $(f \circ g)(\pi/2) + (g \circ f)(\pi)$?

- (a) 0
- (b) -1
- (c) 3
- (d) 1
- (e) 2
- (f) none of the other answer choices provided

8. If $y = 6x - 1$ and $y = bx + 3$ are the equations of perpendicular lines, then what is the value of b ?

- (a) -6
- (b) $1/6$
- (c) $-1/6$
- (d) 6
- (e) 8
- (f) none of the other answer choices provided

9. Find the value of the expression

$$\cos(\tan^{-1}(-\frac{3}{2})).$$

- (a) $2\sqrt{13}/13$
- (b) $-3\sqrt{13}/13$
- (c) $-2\sqrt{13}/13$
- (d) $\sqrt{13}/13$
- (e) $-3/13$
- (f) none of the other answer choices provided

10. Find the value of the expression $\cot(\cos^{-1}(12/13))$.

- (a) $9/5$
- (b) 2
- (c) 1
- (d) $12/5$
- (e) $11/5$
- (f) none of the other answer choices provided

11. Evaluate the following trigonometric expression.

$$\frac{\cos^2(\pi/7) + \sin^2(15\pi/7) - 1}{\sin(\pi/8) \sin(\pi/6) + \cos(\pi/4)}$$

- (a) 1
- (b) 2
- (c) 0
- (d) 3
- (e) 4
- (f) none of the other answer choices provided

12. Find the value of $\sqrt{3} \cos(15^\circ) - \sin(15^\circ)$.

- (a) 1
- (b) $\sqrt{3}$
- (c) $\sqrt{2}$
- (d) $3\sqrt{2}$
- (e) -1
- (f) none of the other answer choices provided

13. Find the value of

$$\frac{\sqrt{1 - \cos(40^\circ)} + \sqrt{1 + \cos(20^\circ)}}{\cos(115^\circ) \cos(25^\circ)}.$$

- (a) $-2\sqrt{6}$
- (b) 2
- (c) $\sqrt{6}$
- (d) $-\sqrt{6}$
- (e) 0
- (f) none of the other answer choices provided

14. Evaluate $\cos(8^\circ) \sin(38^\circ) - \cos(82^\circ) \sin(52^\circ)$.

- (a) $-1/2$
- (b) $\sqrt{2}/2$
- (c) $\sqrt{3}/2$
- (d) $1/2$
- (e) $-\sqrt{3}/2$
- (f) none of the other answer choices provided

15. Given $x + y + z = 90^\circ$. Find $\cos(x) \cos(y) - \sin(x) \sin(y)$.

- (a) $-\sin(z)$
- (b) $\cos(z)$
- (c) $\sin(z)$
- (d) $-\cos(z)$
- (e) $\tan(z)$
- (f) none of the other answer choices provided

16. What is the largest possible value of $3 \sin(x) + 4 \cos(x)$?

- (a) 5
- (b) 7
- (c) 6
- (d) 8
- (e) 9
- (f) none of the other answer choices provided

17. The parabola $y = px^2 + x - 2$ has a vertex on the x -axis. Find p .

- (a) $-1/16$
- (b) 0
- (c) 1
- (d) $-1/8$
- (e) $1/2$
- (f) none of the other answer choices provided

18. Find the exact value of

$$\cos \left[\tan^{-1} \left(\frac{12}{5} \right) + \cot^{-1} \left(\frac{5}{12} \right) \right].$$

- (a) $-119/169$
- (b) $5\sqrt{13}/169$
- (c) $50/169$
- (d) $-5\sqrt{13}/169$
- (e) $119/169$
- (f) none of the other answer choices provided

19. If

$$\frac{-3}{\sin(x)} = \frac{4}{\cos(x)}$$

and $0 < x < \pi$, find the exact value of $\cos(x) - \tan(x)$.

- (a) $1/20$
- (b) $-1/20$
- (c) $9/20$
- (d) $23/20$
- (e) $-3/20$
- (f) none of the other answer choices provided

20. Find the number of solutions of the equation $\cos^2(x) - 8\sin(x) - 8 = 0$ on $0 < x < 2\pi$.

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5
- (f) none of the other answer choices provided

21. Simplify the following expression

$$\frac{(\tan(x) + 1)(\tan(x) + 1) - \sec^2(x)}{\tan(x)}$$

- (a) $2\sin(x)$
- (b) 1
- (c) 2
- (d) $\tan(x)$
- (e) -1
- (f) none of the other answer choices provided

22. Simplify

$$2 + \frac{1}{\cos^2(x)} - \tan^2(x)$$

- (a) 3
- (b) $2 \sin(x)$
- (c) $2 \cos(x)$
- (d) $2/5$
- (e) 1
- (f) none of the other answer choices provided

23. Which of the following is the period (in radians) of the function

$$f(x) = 3 \cot^4(2x - 15^\circ) + 1?$$

- (a) $\pi/2$
- (b) $3\pi/2$
- (c) π
- (d) $15\pi/2$
- (e) 6π
- (f) none of the other answer choices provided

24. Consider the equation

$$\frac{\cot(x) + \frac{\cos^3(x) + \sin^2(x) \cos^3(x)}{1 - \sin^4(x)} - \cos(x)}{\sin(x)} = 0.$$

Find the number of the solutions of the equation.

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 0
- (f) none of the other answer choices provided

25. Consider $\triangle ABC$ with $\angle CBA = 135^\circ$, $|BC| = 2\sqrt{2}$ in, $|AB| = 3$ in. Find the area of the triangle.
- (a) 3 in^2
 - (b) 9 in^2
 - (c) 6 in^2
 - (d) 2 in^2
 - (e) 12 in^2
 - (f) none of the other answer choices provided
26. Consider $\triangle ABC$ with $\angle A = 75^\circ$, $\angle B = 60^\circ$ and $|AC| = 8$ in. Find $|AB|$.
- (a) $4\sqrt{6}/3$ in
 - (b) $8\sqrt{3}/6$ in
 - (c) $\sqrt{6}$ in
 - (d) 8 in^2
 - (e) $8\sqrt{6}/3$ in
 - (f) none of the other answer choices provided
27. Consider $\triangle ABC$ with $\angle B = \angle C + 90^\circ$, $|AC| = 4$ in and $|AB| = 3$ in. What is $\cot(C)$?
- (a) $4/3$
 - (b) $5/3$
 - (c) 1
 - (d) 2
 - (e) 5
 - (f) none of the other answer choices provided

28. Find the smallest solution for the equation $\sin\left(2x - \frac{\pi}{3}\right) + \sin(x) = 0$ on $[0, 2\pi)$.

- (a) $\pi/9$
- (b) 0
- (c) $\pi/8$
- (d) $\pi/6$
- (e) $2\pi/9$
- (f) $8/\pi$
- (g) none of the other answer choices provided

29. $x = \ln(2)$ and $y = \ln(3)$. What is $\log_6(12)$ in terms of x and y ?

- (a) $x/(2y)$
- (b) $(2 + y)/y$
- (c) $(2x + y)/(x + y)$
- (d) $2x/y$
- (e) $(x + 2)/(2y)$
- (f) none of the other answer choices provided

30. Find the radius of the circle $x^2 + y^2 + 12x - 14y + 84 = 0$.

- (a) 6
- (b) 3
- (c) 1
- (d) 8
- (e) 5
- (f) none of the other answer choices provided

31. A rectangular board has perimeter of 72 ft and an area of 288 ft^2 . Find the width of the rectangle.
- (a) 24 ft
 - (b) 10 ft
 - (c) 12 ft
 - (d) 14 ft
 - (e) 11 ft
 - (f) none of the other answer choices provided

32. If $\vec{a} = \vec{i} + 2\vec{j}$ and $\vec{b} = 3\vec{i} + 3\vec{j}$, what is the the magnitude of the vector

$$\frac{1}{3}\vec{b} - \vec{a}?$$

- (a) 1
 - (b) 2
 - (c) 3
 - (d) 8
 - (e) 5
 - (f) none of the other answer choices provided
33. Find the number of points where the circle $x^2 + y^2 = 100$ and the parabola $x^2 + 2y = 1$ intersect.
- (a) 0
 - (b) 3
 - (c) 1
 - (d) 2
 - (e) 6
 - (f) none of the other answer choices provided.

34. The graph of the equation $y = -|x| - 6$ is symmetric with respect to
- (a) the x -axis
 - (b) the y -axis
 - (c) the origin
 - (d) the x -axis and the y -axis
 - (e) the x -axis and the origin
 - (f) none of the other answer choices provided.

35. If

$$ax^3 + bx^2 + cx + d = (x^2 - 1)(mx + n) + 3x + 4,$$

what is $b + d$?

- (a) 4
 - (b) 1
 - (c) 2
 - (d) 5
 - (e) 3
 - (f) none of the other answer choices provided.
36. How many solutions does $4 \cos^2(\theta) - 3 = 0$ have on $0 \leq \theta < 2\pi$?
- (a) 3
 - (b) 2
 - (c) 0
 - (d) 4
 - (e) 1
 - (f) none of the other answer choices provided

37. Identify the graph of the equation $4x^2 - 4xy + 5y^2 - 20 = 0$.

- (a) ellipse
- (b) parabola
- (c) hyperbola
- (d) circle
- (e) line
- (f) none of the other answer choices provided

38. Identify the type of conic given by

$$4x^2 + 8xy + 4y^2 - 2x - 5y - 2 = 0.$$

- (a) parabola
- (b) ellipse
- (c) hyperbola
- (d) circle
- (e) line
- (f) none of the other answer choices provided

39. Find the x -coordinate of the center of the circle $x^2 + y^2 - 6x + 14y + 33 = 0$.

- (a) -7
- (b) 5
- (c) -3
- (d) 3
- (e) 1
- (f) none of the other answer choices provided

40. Find the y -coordinate of the vertex of the parabola $f(x) = 2(x - 4)^2 - 5$.

- (a) -5
- (b) 5
- (c) 4
- (d) 2
- (e) 1
- (f) none of the other answer choices provided

41. Find the radius of the circle

$$(x + 3)^2 + \left(y - \frac{1}{3}\right)^2 = 11.$$

- (a) 11
- (b) 121
- (c) $\sqrt{121}$
- (d) 12
- (e) $\sqrt{11}$
- (f) none of the other answer choices provided

42. If

$$\frac{\tan(4x) - \tan(3x)}{1 + \tan(4x)\tan(3x)} = \frac{1}{2},$$

what is the value of $\tan(2x)$?

- (a) $4/3$
- (b) $3/4$
- (c) $-3/4$
- (d) -1
- (e) 1
- (f) none of the other answer choices provided

43. Which of the following is **not** a solution of the equation $\tan(2\theta) + 2\cos(\theta) = 0$ on the interval $0 \leq \theta < 2\pi$?

(a) $\pi/2$

(b) $7\pi/6$

(c) $\pi/6$

(d) $3\pi/2$

(e) 3

(f) none of the other answer choices provided

Correction: Answer choice (e) should be omitted. The contest results were not affected by choice (e) being included.

44. The vector $\vec{v} = 4\vec{i} - 4\vec{j}$. Find the direction angle of \vec{v} .

(a) 215°

(b) 45°

(c) 60°

(d) 30°

(e) 315°

(f) none of the other answer choices provided

45. Find the solution set of the equation $\tan^{-1}(x^2 + x - 12) = -\pi$.

(a) $\{-4, 3\}$

(b) $\{-5, 6\}$

(c) $\{-5, 3, 6\}$

(d) $\{-5, 2, 6\}$

(e) $\{-5, 3, 6\}$

(f) none of the other answer choices provided.

46. Find the solution of the equation $\tan^{-1}(2x + 1) = \cot^{-1}(5)$.

- (a) $-1/5$
- (b) $-1/8$
- (c) $1/5$
- (d) $-2/5$
- (e) $2/5$
- (f) none of the other answer choices provided.

47. One of the vertical asymptotes of

$$y = \frac{2x^2}{2x^2 - x - 3}$$

is

- (a) $x = 1$
- (b) $x = -1$
- (c) $x = -3/2$
- (d) $x = 3$
- (e) $x = 4$
- (f) $x = 2/3$
- (g) none of the other answer choices provided.

48. $y = 6x^4 + 7|x|$ is

- (a) an even function
- (b) an odd function
- (c) neither an odd nor an even function
- (d) a trigonometric function
- (e) a polynomial function
- (f) a linear function
- (g) none of the other answer choices provided.

49. Simplify $\sin(x + 2\pi) \cot(x + \pi)$.

- (a) $\sin(x)$
- (b) $\cot(x)$
- (c) $\cos(x)$
- (d) $\tan(x)$
- (e) $\cos(2x)$
- (f) none of the other answer choices provided.

50. Simplify

$$\frac{\cos^2(a) - \cos^2(b)}{\sin(a + b) \sin(a - b)}$$

- (a) -1
- (b) $\tan(a)$
- (c) $\tan(2a)$
- (d) 1
- (e) $\tan(3a)$
- (f) none of the other answer choices provided.