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# Calculator Exam

1. Reduce the fraction $\frac{3424175370}{19908504}$.

| **Answer:** | |
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2. Give the sum of the prime factors of $5617831492$.

| **Answer:** | |
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3. Let $a_n = n^2 - 2n + 3$ for each natural number *n*. Find $a_1 - a_2 + a_3 - a_4 + \cdots + a_{49} - a_{50}$.

| **Answer:** | |
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4. Let $f(x) = 1 + x\left(2 - x\left(3 - x\left(4 - x(1+x)\right)\right)\right)$. Find the solution to $f\left(f(x)\right) = \frac{1}{2}$ which is closest to $x = 0$.

| **Answer:** | |
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5. Let $f(x) = 1 + x\left(2 - x\left(3 - x\left(4 - x(1+x)\right)\right)\right)$. Give the sum of the negative solutions to $f\left(f(x)\right) = \frac{1}{2}$ as a decimal number rounded to 8 decimal places.

| **Answer:** | |
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6. Factor $x^8 - 8x^7 - 2x^6 + 184x^5 - 587x^4 + 472x^3 + 516x^2 - 720x$.

| **Answer:** | |
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7. Give an approximation for the largest value of the function $f(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{1}{10}\left(1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24}\right)$ for $-2 \le x \le 2$ which is accurate to 8 decimal places.

| **Answer:** | |
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8. Round the number $\left(1-\frac{1}{4}\right)^2\left(1-\frac{1}{9}\right)^3\cdots\left(1-\frac{1}{10000}\right)^{100}$ to 8 decimal places.

| **Answer:** | |
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9. A line passes through the point (13,87). A second line intersects this line at the point (-21,37) and has *y* intercept -43. Give the *x* coordinate of the *x* intercept of the second line in fraction form.

| **Answer:** | |
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10. Give the number of solutions to $(x+1)(x+2)(x-3)(x-5)=\frac{1}{x}+\sqrt{1-x}$ .

| **Answer:** | |
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11. Suppose your grandmother deposited \$100,000 in an account on January 1, 2006 which earns 0.25% interest on the last day of every month. She has told you that you can withdraw small amounts of money from this account. So, you decide to withdraw \$500 from this account on the last day of every month (once you have seen that the interest from the month has been recorded into the account). What will be the last date on which you can withdraw this amount of money from the account?

| **Answer:** | |
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12. The numbers -200 to 200 are arranged in order along a line. A person moves from left to right along the list of numbers, examines each number, performs an operation, and then moves to the next number to the right. If the number is even, she exchanges the position of the number in the list with the first occurance to the right of the negative of this number. If the number is odd, divisible by 3, and smaller than 197, then she adds 1 to the number and exchanges it with the number 3 positions to the right. She ignores all other numbers. After she has completed her task, what are the 237$^{th}$, 238$^{th}$, 239$^{th}$ and 240$^{th}$ numbers in the list.

| **Answer:** | |
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13. Approximate $\sqrt{\left|\left|\left|\left|\left|-\frac{13}{3}-\frac{41}{89}\right|-\frac{11}{53}\right|-\frac{2}{3}\right|-\frac{7}{17}\right|+2\right|+\frac{21}{13}-7.3+4}$ to 8 decimal places

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| **Answer:** | |

14. Add the sum of the first 2 positive integer multiples of 2, to the sum of the first 3 positive integer multiples of 3, to the sum of the first 4 positive integer multiples of 4, and so forth, up to the sum of the first 20 positive integer multiples of 20. Then subtract the sum of the first 300 odd integers and divide by 21. Give the result to 8 decimal places.

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| **Answer:** | |

15. $s_1 = 1$, $s_2 = s_1 + \frac{1}{2+s_1}$, $s_3 = s_2 + \frac{1}{3+s_2}$, … , $s_{50} = s_{49} + \frac{1}{50+s_{49}}$. Approximate the value of $s_1 + s_2 + s_3 + \cdots + s_{50}$ to 8 decimal places.

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| **Answer:** | |

16. Determine the number of fractions in reduced form *m*/*n* where *m* and *n* are integers so that $1 < m < n < 20$, and *m* is divisible by either 2, 3 or 5.

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| **Answer:** | |

17. Round the number $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots+\frac{1}{10101}$ to 8 decimal places.

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| **Answer:** | |

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18. **Tie-Breaker.** If you flip a fair coin several times, there is a chance that your flip will turn up heads many times in succession. Suppose you decide to simulate this process on your calculator by writing a short program that flips a fair coin 100 times in succession and keeps track of the longest streak of either consecutive flips of heads or consecutive flips of tails, and records this at the end of the simulation. Then you run this simulation multiple times. From your observations, what is your best estimate of the probability that the longest streak will be 8?

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| **Answer:** | |