Name: _____

School: _____

University of Houston
High School Contest – Spring 2006
Geometry Test

Directions:
You have 50 minutes to complete this exam. Calculators are not permitted. You must write your answer in the answer blank provided. Answers should be exact (such as $\frac{16\pi}{5}$, $2\sqrt{3}$) and should be written in simplest form. If units are given in the question, then the answer should be written with appropriate units.

1. Find the perimeter of the following swimming pool. Assume that all intersecting sides in the diagram are perpendicular to each other.

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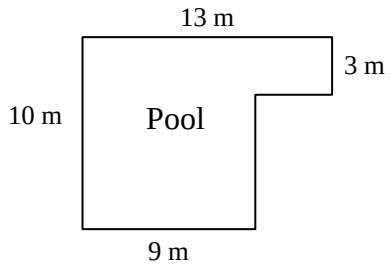

2. $\overleftrightarrow{CD}$ bisects $\overline{AB}$ at point E. If $AE = 2x + 6$ and $AB = 6x - 12$, find the length of $\overline{AB}$.

_____

3. In ΔGEO, the measure of $\angle G$ is 62^o and the measure of $\angle O$ is 58^o. List the three sides of the triangle in order from the least to greatest measure.

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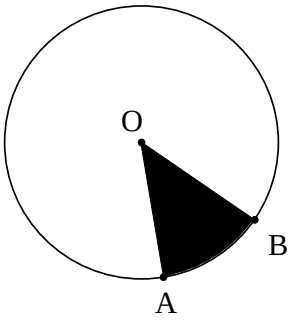
4. Circle O is shown below and has radius 6 cm. If the length of $\overset{\frown}{AB}$ is π cm, find the area of the shaded sector.



5. If quadrilateral *MATH* is inscribed in a circle and the measure of angle *M* is 71^{o}, find the measure of angle *T*.

6. A regular polygon has side length 8 cm, and the angle between two adjacent sides is 120°. Find the area of the polygon.

7. Write the converse of the following statement and then state whether the converse is true or false:
If a hexahedron is a prism, then it has twelve edges.

Converse:

_____

_____

Is the converse true or false? _____

8. A triangle has vertices $A(-1,-3)$, $B(7,1)$, and $C(6,5)$, and median $\overline{CD}$ is drawn from vertex *C*. Find the length of median $\overline{CD}$.

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9. In the diagram below, circle A has radius 10 inches. If $\overline{CD} \perp \overline{AB}$, and the length of $\overline{DB}$ is 4 inches, find the length of $\overline{CD}$.

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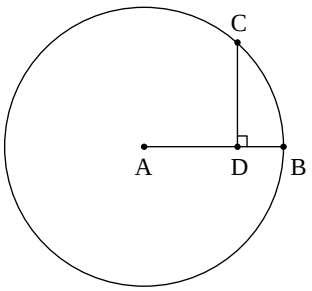
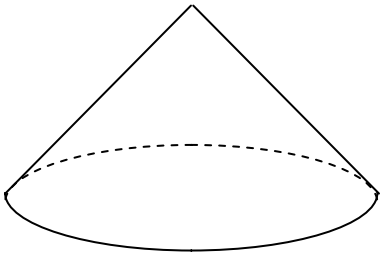
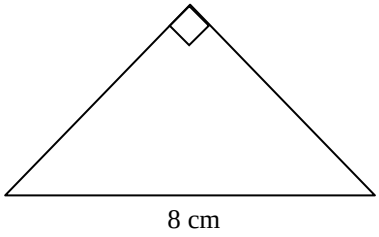


10. Consider the right circular cone shown below. A cross-section of the cone is shown which is perpendicular to the base, and passes through the center of the base. Find the total surface area of the cone.

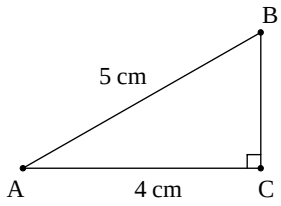


Right Circular Cone

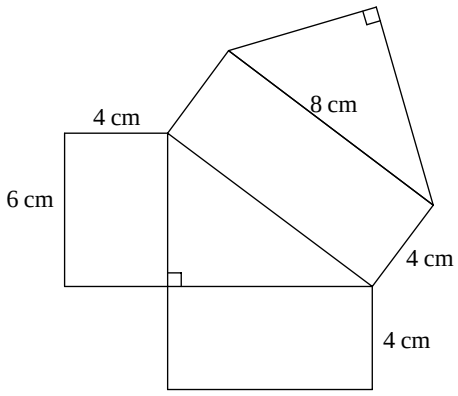


Cross-Section Perpendicular
to the Base
(passes through the center of the Base)

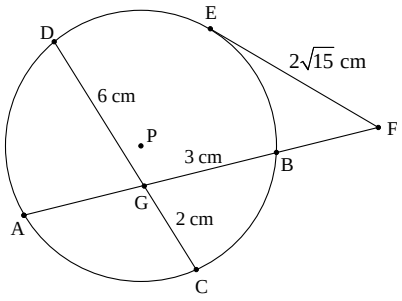
11. Using triangle *ABC* below, find the value of: $\cos A - \tan B$.



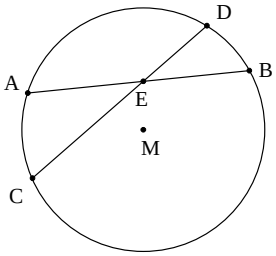
12. The net below is comprised of three rectangles and two congruent right triangles. Find the volume of the solid formed from this net.



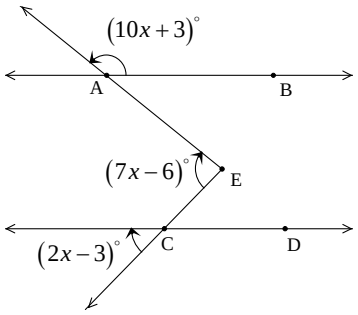
13. In circle *G* below, chords $\overline{AB}$ and $\overline{CD}$ intersect at point *G*. Secant $\overline{AF}$ and tangent $\overline{EF}$ are also shown. Find the length of $\overline{BF}$.
(In the diagram, $DG = 6$, $GC = 2$, $GB = 3$, and $EF = 2\sqrt{15}$, and all units are in centimeters.)



14. In the figure below, $\overline{AB}$ and $\overline{CD}$ are chords of circle *M* which intersect at point *E*. The degree measure of $\overset{\frown}{AC}$ is eight less than three times the degree measure of $\overset{\frown}{DB}$, and the measure of $\angle DEB$ is 24^o. Find the degree measure of $\overset{\frown}{AC}$.



15. In the figure below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Find the value of *x*.



16. In ΔABC, $AB = 10$ and $BC = 21$. The length of side $\overline{AC}$ must be between what two measurements? (Write the two answers in the blanks provided below.)

____ < AC < ____

17. Consider the region below formed by three semicircles with diameters $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, where point B lies on the semicircle defined by diameter $\overline{AC}$. Find the area of the shaded region.

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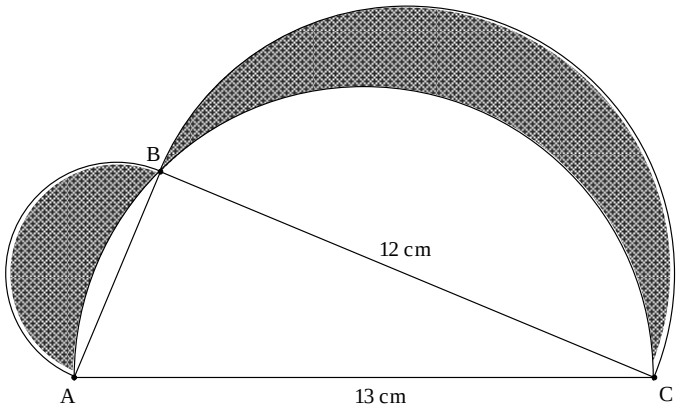

18. Find the area of a regular octagon with side length 10 cm.

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19. Circle A and Circle B are externally tangent and have radii of 4 cm and 10 cm, respectively. If an external tangent is drawn to the two circles that intersects circle *A* at point *C* and intersects circle *B* at point *D*, find the length of $\overline{CD}$.

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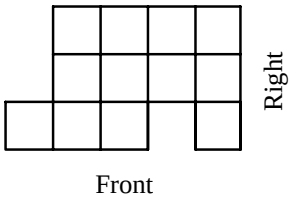
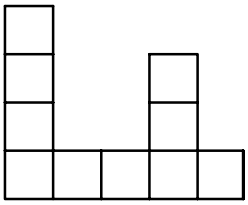
20. A building is made from stacked cubes, and views from five different perspectives are illustrated below. How many cubes are needed to construct this building?

_____

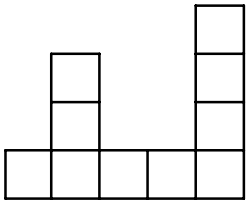
Top View:



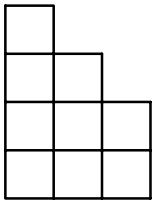
Front View:



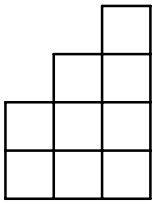
Back View:



Right View:



Left View:



21. An equilateral triangle has 24 cm added to its perimeter to make a new equilateral triangle. If the area of the new equilateral triangle is three times the area of the initial equilateral triangle, find the length of a side of the new equilateral triangle.

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22. The width, length, and height of a rectangular box are in the ratio of $2\sqrt{5}:5:6$. If the diagonal of the box is 18 inches long, find the volume of the box.

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23. Three ounces of water are placed into a right circular cone, as shown below. How many more ounces of water must be added to double the height of the water?

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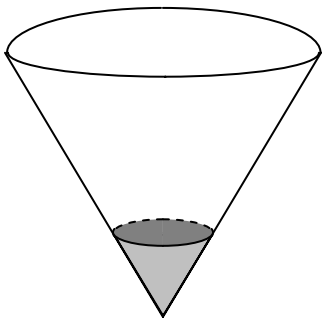

24. A silo is composed of a right circular cylinder topped by a hemisphere, both having the same radius. If the volume of the silo is 90π cubic meters, and the diameter of the silo is 6 meters, find the height of the silo.

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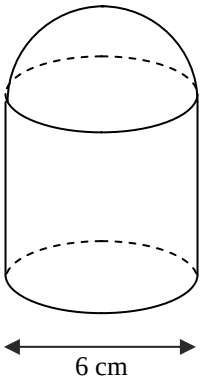

25. Give the equation of the line passing through the origin that divides rectangle EFGH into two regions having equal areas. (Write the equation in slope-intercept form.)

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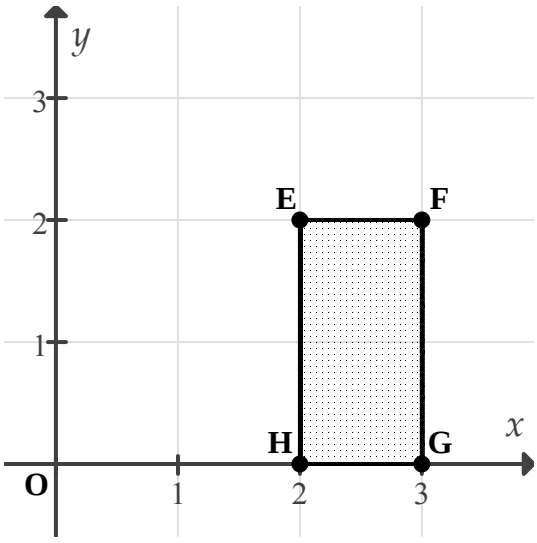

26. A semicircle with diameter 8 cm is cut out of paper and rolled into a cone, where the endpoints of the diameter are connected and the circle's center becomes the vertex of the cone. Find the volume of the cone.

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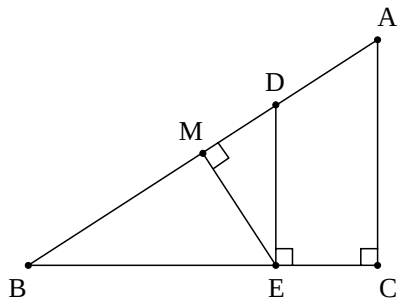
27. In right triangle *ABC* shown below, *M* is the midpoint of $\overline{AB}$, $\overline{DE} \perp \overline{BC}$, and $\overline{EM} \perp \overline{AB}$. If the length of $\overline{AC}$ is 6 cm and the length of $\overline{BC}$ is 8 cm, find the length of $\overline{BD}$.



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28. Two cylindrical cups are similar and are used to hold ice cream. Cup A holds five scoops of ice cream when filled to the top and packed down, and cup B holds eight scoops of ice cream when filled to the top and packed down. Find the ratio of the lateral area of cup A to the lateral area of cup B. (Assume that all scoops are the same size.)

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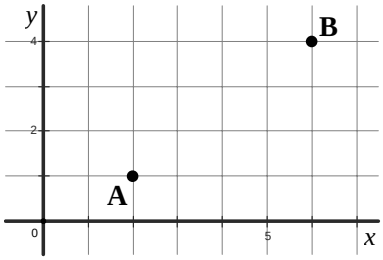
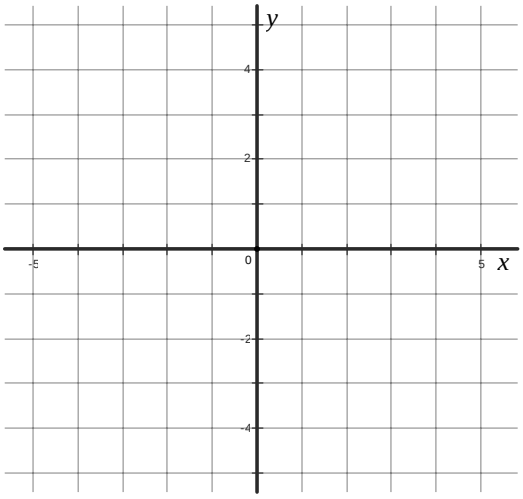
29. In Taxicab Geometry, the shortest distance between two points is defined as the minimum number of city blocks a taxi would need to travel to get from one point to the other, assuming square blocks of equal size and all streets oriented only horizontally and vertically. In the diagram below, for example, the distance between the points $A(2,1)$ and $B(6,4)$ is 7.



Using this Taxicab definition of distance, use the grid below to draw a Taxicab circle of radius 4, centered at the origin. (Use the definition that a circle is the set of all points whose distance to the center is the radius.)

Answer on the grid below:



30. Suppose that a square is divided into nine smaller squares of equal size and the center piece is removed (shown in black), as depicted below.

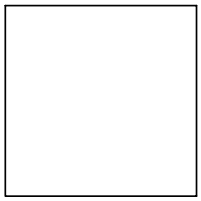


Figure 1

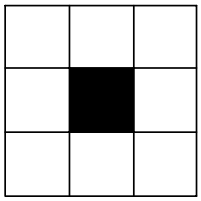


Figure 2

This process is repeated again with Figure 2, yielding Figure 3 below.

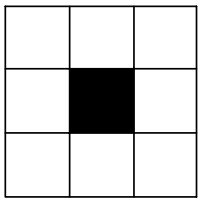


Figure 2

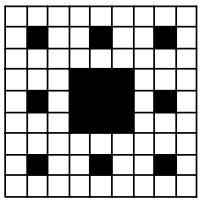


Figure 3

Suppose that the process is repeated yet again with Figure 3, yielding a diagram called Figure 4 (not shown). If a dart is thrown randomly at Figure 4 and can land on any point within Figure 4 with equal probability, what is the probability that the dart will land on a black square?

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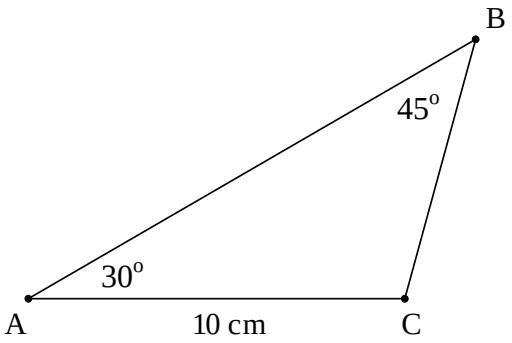
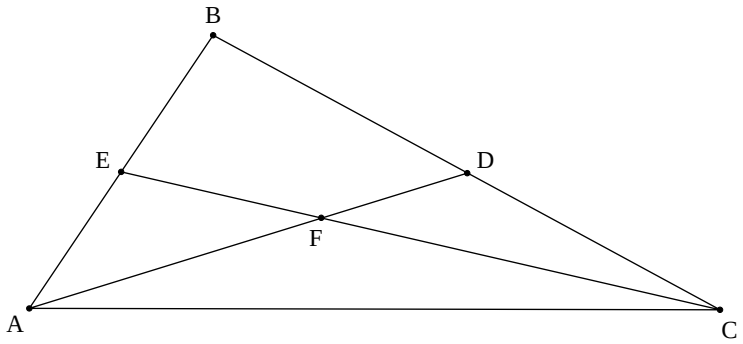
31. Find the length of $\overline{BC}$ in the diagram below.



32. Triangle *ABC* is shown below with medians $\overline{AD}$ and $\overline{CE}$, which intersect at point *F*. If the length of $\overline{AD}$ is 9 cm, the length of $\overline{CE}$ is 12 cm, and the measure of $\angle EFA$ is 30^o, find the area of quadrilateral *EBDF*.



33. A, B, C, and D are points on the circle below and $\overline{CD}$ is a diameter of the circle. The length of $\overline{DB}$ is 4 cm, the length of $\overline{BC}$ is 8 cm, and $\angle DBA \cong \angle ABC$. Find the length of $\overline{AB}$.

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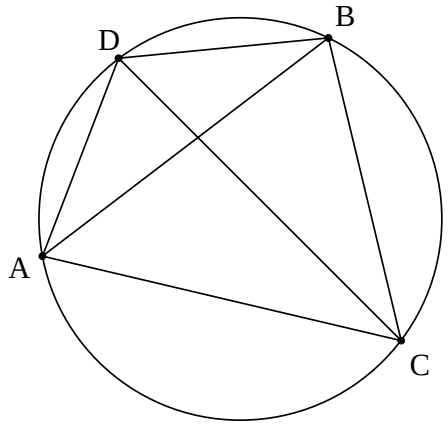

END OF EXAM ☺